reader reflections

reactions to articles and points of view on the teaching of mathematics

No time sharing

I read "A Summer Course with the TI 57 Programmable Calculator," by Eli Maor (February 1980), with a mixture of delight and envy.

Having been introduced to the TI 57 through a National Science Foundation program at the University of Dallas, I became convinced that it was a viable alternative to time-sharing systems or minicomputers in the classroom. As a consequence of my conviction, we dropped our long-standing, time-sharing contract and purchased eighteen TI 57s for my eighth-grade programming course. The benefits have been significant. Waiting time, previously necessitated by limited terminal facilities, has been cut to zero. Course time, previously spent in teaching the intricacies of formatting, is now spent in teaching mathematical concepts associated with more advanced programming. Finally, our budget, a source of concern to every administrator, has been drastically reduced to reflect current schoolwide needs, but because of the nominal cost of the TI 57, our program has not suffered.

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Projects change attitudes

A few years ago I started requiring my geometry students to complete a project as part of the class assignment. The purpose is to make students aware of a need for mathematics in areas of their own interest.

The project assignment along with a bibliography of available resource books is given about the third week of the school year. The projects are due on or before the Monday following Thanksgiving. The rules are as follows:

1. Each student must turn in a project.
2. Each student must be able to describe the project's relationship to mathematics.
3. The project should relate to an area of interest of the student.
4. The project is to be done outside class and must be the student's own choice.

I will listen to any problems as they arise and advise when necessary.

Over a number of years I have accumulated an enviable collection of projects. Some have been as simple as a candle formed into a cylinder. Songs have been written. Art designs created. Even plans for a house designed by using the Pythagorean theorem as a starting point have been submitted.

Not only do the students learn from their own research but they are much more aware of the uses of mathematics in this course than in the traditional course. Their efforts relieve some pressure they might feel in a rigorous study of geometry. The grading is subjective because of the nature of the task.

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For some related ideas on projects in this issue see "Sharing Teaching Ideas."—*Ed.*

A corollary

I read with interest the article "Nick's First Theorem" (April 1980). I regret to inform the authors that their "theorem" is a special case of my factorization theorem. I have assuredly found an admirable proof of this, but the margin is too narrow to contain it.

If you have any questions, please feel free to call me.

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The authors respond: If one considers the equation

$$x^n + y^n = z^n$$

where x, y, and z are integers, we have been able to prove that for all integral values of n, Craig Laferty does not exist.

Word problems

I read with interest the examples of word problems presented by Hal Saunders, "When Are We Ever Gonna Have to Use *This?*" (January 1980). It reminded me of a problem that was given to me by a supervisor from a plant that employs sheet-metal workers.

A computer-driven laser beam is used in this plant to cut patterns from sheet metal and to produce the bolt holes that are used to assemble the pieces. The cutting device operates by first imposing a coordinate system on the metal. Then the laser produces small holes at coordinates that are specified by the computer. A pattern is cut out by making a lot of holes very close together along the cut line. Most of the curves used in practice are combinations of straight line segments and arcs (of circles). The metal engineer determines how many holes to make and where to put them. He must give the computer instructions, usually a set of formulas, for locating the coordinates.

Problem 1. How many holes must be made to cut out a circle 4 inches in diameter if the holes are to be equally spaced, with centers 1/16 inch apart?

Problem 2. Give a formula for the coordinates of 10 holes (bolt holes), equally spaced, each of whose centers lie 6 inches from a fixed point. (Assume the fixed point is assigned to (0, 0).)

Answers: (1) 202 holes (a fractional hole counts as a hole); (2) If we assume that one hole is located at (6, 0), the 10 holes have coordinates (x, y) where $x = 6\cos 36n°$ and $y = 6\sin 36n°$ for $n = 0, \ldots, 9$.

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The fuss about trapezoids

In the May 1980 issue there were two letters objecting to Stephen Maraldo's approach to teaching properties of quadrilaterals (January 1980). The discussion centers on the way *trapezoid* should be defined. According to Maraldo, a trapezoid can have one or two pairs of parallel sides; the objectors feel that this is incorrect—that *trapezoid* should be defined to have one and only one pair of parallel sides. This discussion prompts me to make the following points, observations, and recommendations.

1. *Certain terminology dies hard, even in times in which much terminology is being redefined.* In the 1960s teachers and mathematicians took a hard look at school mathematics from many points of view, and as a result many changes were made, among them many changes in terminology. Following are a few examples.

a) A *polynomial,* in high school books, had been defined as an expression with more than two terms.

b) *Parallel lines* had been defined in some books to be lines that are everywhere equidistant.

c) An *angle* had been defined, à la Euclid, as the *inclination* of one line to another.

A *trapezoid* had been defined to a quadrilateral with one and only one pair of parallel sides. Most authors of the mid sixties adopted a more general definition, allowing a trapezoid to have one or two pairs of parallel sides.

The newer and better definitions that had been suggested were accepted almost universally, but not the definition of trapezoid! I coauthored several texts in which the more general definition was used, and as a result I received a high volume of mail bringing me to task on this point. Eventually I wrote an article "What Is a Trapezoid?" (November 1966). The mail did not stop, but the volume dwindled. Some authors of mathematics texts went back to the old definition, apparently to silence the critics. Others, including myself, stuck to our guns.

2. ***Human beings can get very emotional about language in general, and about points of terminology in particular.*** **I have observed and been involved in many discussions respecting mathematics terminology. I have heard what might be called "debates" over which of two terms would be better, debates in which at least some of the participants tried to put forth logical reasons for making a choice. The results are almost always the same—people hold onto their preferences for terminology according to their prejudices; they are not convinced by rational arguments. I expect that few readers who prefer the older definition of trapezoid will be persuaded by my arguments.**

3. *Many people do not realize that definitions in mathematics are arbitrary.* It is not uncommon to find two mathematicians using incompatible definitions. In a particular discourse, a mathematician defines the terms and then proceeds, and if definitions have been made differently elsewhere this doesn't worry another mathematician. The psychological and pedagogical problems that can arise if terms are not consistently defined for children should not be minimized, but we would hope that the teacher of secondary school mathematics is enough of a mathematician to know that one does not *go to the dictionary* to find definitions of mathematical terms! Unfortunately, this is not the case. Many of the letters I received quoted dictionary sources.

One would also hope that secondary teachers would know that Euclid-worshiping has been out of style for some time, and that it is no longer in vogue to look to Euclid to settle questions of terminology. This is not the case, either, as one can see by reading Ken Seydel's letter (May 1980).

4. *Not enough thought is given to certain aspects of desirable definitions.* A good definition should be reversible, but attention should also be given to some of the more aesthetic qualities of a good mathematical definition. Some of them are as follows.

a) *Generality.* When there is a choice between two ways of making a definition, as in the case of a trapezoid, it is almost always better to choose the more general one, because a certain economy is gained by so doing. If *trapezoid* is defined so that a parallelogram is a special kind of trapezoid, any statement proved about trapezoids will also hold for parallelograms and need not be proved again. For example, any formula for the area of a trapezoid is immediately applicable to a parallelogram. If it is proved that the diagonals of a trapezoid divide each other proportionally in the same ratio as the bases, then we know that the same is true of a parallelogram, and thus that the diagonals of a parallelogram bisect each other.

Maraldo's article seems to result from a desire to use the advantages of generality, and in fact his diagram and accompanying explanation are a fine argument in favor of the use of the more general definition.

b) *Consistency of philosophy.* The generality just mentioned occurs in many of our definitions. For example, an equilateral triangle is a special kind of isosceles triangle, a square is a special kind of rectangle, and so on. (This was not always the case.) For consistency of philosophy, the more general definition