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A remark on representable positive cylindric algebras

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A *positive* cylindric algebra is a positive reduct of a CA, i.e., an algebra, denoted $\mathfrak{Rd}_+\mathfrak{A}$, obtained from a cylindric algebra $\mathfrak{A}$ by discarding the complementation operation from the fundamental operations of $\mathfrak{A}$. Let $\mathfrak{A}(\alpha, U)$ denote the full cylindric set algebra with base U and dimension α. In analogy with definitions for cylindric algebras we define the class Cs_α^+ of *positive set algebras* of dimension α as $Cs_\alpha^+ = S\{\mathfrak{Rd}_+\mathfrak{A} : \mathfrak{A} \in Cs_\alpha\}$ $(= S\{\mathfrak{Rd}_+\mathfrak{A}(\alpha, U) : U \neq \emptyset\})$ and the class RCA_α^+ of *representable positive cylindric algebras* of dimension α as $RCA_\alpha^+ = SP(Cs_\alpha^+)$.

Representable positive cylindric algebras arise in a natural way in the study of databases, cf., [1], [3]. In this note we observe that the nonfinite axiomatizability of RCA_α, for $\alpha \geq 3$, established by Monk [5] (cf., section 4.1 of [3]), extends to RCA_α^+. This answers a question posed to the author by W. Lipski in 1983.

LEMMA 1. *Suppose* $\mathfrak{A}$ *and* $\mathfrak{B}$ *are* CA_α*'s.*

(i) *If* f *is a homomorphism of* $\mathfrak{Rd}_+\mathfrak{A}$ *into* $\mathfrak{Rd}_+\mathfrak{B}$, *then* $f : \mathfrak{A} \to \mathfrak{B}$ *is a* CA_α*-homomorphism.*

(ii) $\mathfrak{Rd}_+\mathfrak{A} \in RCA_\alpha^+$ *if and only if* $\mathfrak{A} \in RCA_\alpha$.

Proof. (i) is obvious since $-x$ is uniquely determined in $\mathfrak{Rd}_+\mathfrak{A}$ by the equations $x + -x = 1$ and $x \cdot -x = 0$.

(ii) For the implication $\Rightarrow$, by 2.4.39 of [2], it suffices to show, for $a \in A$, $a \neq 0$, there exist a set $U \neq \emptyset$ and a homomorphism $f : \mathfrak{A} \to \mathfrak{A}(\alpha, U)$ with $fa \neq \emptyset$. Since $\mathfrak{Rd}_+\mathfrak{A}$ is a subdirect product of positive set algebras, there exist U and $f : \mathfrak{Rd}_+\mathfrak{A} \to \mathfrak{Rd}_+\mathfrak{A}(\alpha, U)$ with $fa \neq \emptyset$. The conclusion follows from (i). The reverse implication $\Leftarrow$ is obvious.

THEOREM 2.

(i) RCA_α^+ *is a universal class for all* α.

(ii) *For* $\alpha \geq 3$ RCA_α^+ *is not finitely scheme axiomatizable.*

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Proof. (i) Since $RCA_\alpha^+ = SP(\mathfrak{Rd}_+ Cs_\alpha) = S\mathfrak{Rd}_+(PCs_\alpha) = S\mathfrak{Rd}_+(RCA_\alpha)$ and RCA_α is an equational class, it follows from Theorem 1.9 of [6] that RCA_α^+ is a universal class.

(ii) First assume $3 \le \alpha \prec \omega$. By the proof of Theorem 1.11 of [5] (or, 4.1.3 of [3]) there exist CA_α's $\mathfrak{A}_\kappa$ $(\kappa \in \omega)$ such that $\mathfrak{A}_\kappa \notin RCA_\alpha$ and $\prod_{\kappa\in\omega} \mathfrak{A}_\kappa/D$ is in RCA_α for a nonprincipal ultrafilter D on ω. By 1(ii), $\mathfrak{Rd}_+ \mathfrak{A}_\kappa \notin RCA_\alpha^+$, but $\prod_{\kappa\in\omega} \mathfrak{Rd}_+ \mathfrak{A}_\kappa/D = \mathfrak{Rd}_+(\prod_{\kappa\in\omega} \mathfrak{A}_\kappa/D)$ is in RCA_α^+. The nonfinite axiomatizability follows. The proof for $\alpha \ge \omega$ is similar using Theorem 2.2 of [5] (or 4.1.7 of [3]).

Similar results hold for positive reducts of polyadic algebras. More generally, if the class of cylindric or polyadic algebras is enriched by definable operations, for example the inner cylindrifications C_i^∂ (cf., [2], 1.4.1), the results hold for the positive reducts of these classes as well.

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