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ON CONNECTIONS BETWEEN INFORMATION SYSTEMS, ROUGH SETS AND ALGEBRAIC LOGIC

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In this note we remark upon some relationships between the ideas of an approximation space and rough sets due to Pawlak ([9] and [10]) and algebras related to the study of algebraic logic — namely, cylindric algebras, relation algebras, and Stone algebras.

The paper consists of three separate observations. The first deals with the family of approximation spaces induced by the indiscernability relation for different sets of attributes of an information system. In [3] the family of closure operators defining these approximation spaces is abstractly characterized as a certain type of Boolean algebra with operators. An alternate formulation in terms of a general class of diagonal-free cylindric algebras is given in 1.6. The second observation concerns the lattice theoretic approach to the study of rough sets suggested by Iwiński [6] and the result by J. Pomykała and J. A. Pomykała [11] that the collection of rough sets of an approximation space forms a Stone algebra. Namely, in 2.4 it is shown that every regular double Stone algebra is embeddable into the algebra of all rough subsets of an approximation space. Finally, a notion of rough relation algebra is formulated in Section 3 and a few connections with the study of ordinary relation algebras are established.

1. Approximation algebras associated with information systems. An *information system* in the sense of Pawlak [9] is a 4-tuple $S = \langle U, \Omega, V, f\rangle$ where U is a set, Ω is a finite set, V is a function with $\operatorname{Dom} V = \Omega$ and $f : U \to \prod_{a\in\Omega} V_a$.

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Each set $P \subseteq \Omega$ determines an equivalence relation θ_P on U, the indiscernibility relation for P, defined for $x, y \in U$ by

$$x\theta_P y \Leftrightarrow \forall a \in P\ (fx)_a = (fy)_a\,.$$

The relation θ_P induces a natural closure operation $\overline{P}$ on subsets of U by

$$\overline{P}(A) = \bigcup\{\theta_P x : x \in A\} \qquad \text{for } A \subseteq U\,.$$

$\overline{P}(A)$ is called the *P-upper approximation* of A. Using these closure operations we form a Boolean algebra with operations

$$\mathcal{B}_S = \langle SbU, \cup, \cap, \sim, \emptyset, U, \overline{P}\rangle_{P\subseteq\Omega}$$

called in [3] the *knowledge approximation algebra of type Ω derived from the information system S*. The following definition was proposed in [3] to axiomatize the class of algebras $\mathcal{B}_S$.

DEFINITION 1.1. An algebra $\mathcal{B} = \langle B, +, \cdot, -, 0, 1, \kappa_P\rangle_{P\subseteq\Omega}$ is a *knowledge approximation algebra of type Ω* if $\kappa_P \in B^B$ for each $P \subseteq \Omega$ and the following axioms hold for all $x, y \in B$ and $P, Q \subseteq \Omega$:

(A_0) $\langle B, +, \cdot, -, 0, 1\rangle$ is a complete atomic Boolean algebra,
(A_1) $\kappa_P 0 = 0$,
(A_2) $\kappa_P x \geq x$,
(A_3) $\kappa_P(x \cdot \kappa_P y) = \kappa_P x \cdot \kappa_P y$,
(A_4) $x \neq 0$ implies $\kappa_\emptyset x = 1$,
(A_5) $\kappa_{P\cup Q} x = (\kappa_P x) \cdot (\kappa_Q x)$ if x is an atom of B.

$\mathcal{B}$ is called *reduced* if $\kappa_\Omega x = x$ for all $x \in B$. Note that $\mathcal{B}_S$ is reduced if and only if f is one-one.

The following result establishes a representation for knowledge approximation algebras which shows the axioms in Definition 1.1 characterize the class of algebras derived from information systems.

THEOREM 1.2 ([3], Theorem 11). *If $\mathcal{B}$ is a reduced knowledge approximation algebra of type Ω with Ω finite, then $\mathcal{B} \cong \mathcal{B}_S$ for some information system S.*

Because the elements of $\mathcal{B}_S$ can be interpreted as sets of Ω-sequences and the operators $\overline{P}$ act like cylindrifications it has been suggested that Theorem 1.2 could be formulated in the language of cylindric algebras. The formulation below was worked out with assistance and prompting by Don Pigozzi.

To formulate the result we need to generalize the notion of diagonal-free cylindric algebra from 1.1.2 of HMT [5]. A complication arises because we need the generalized cylindrifications $c_{(\Gamma)}$. However, since we drop the cylindric axiom (C_4) the usual definition of a generalized cylindrification does not give a well-defined operation. For this reason the similarity type of our algebras includes an operation for each subset of Ω.

DEFINITION 1.3. An algebra $\mathcal{B} = \langle B, +, \cdot, -, 0, 1, c_\Gamma \rangle_{\Gamma \subseteq \Omega}$ is a *strong noncommutative diagonal-free* CA_Ω, strong $NCDf_\Omega$ for short, if for all $x, y \in B$ and for all $\Gamma \subseteq \Omega$,

(C_0) $\langle B, +, \cdot, -, 0, 1 \rangle$ is a Boolean algebra,
(C_1) $c_\Gamma 0 = 0$,
(C_2) $c_\Gamma x \geq x$,
(C_3) $c_\Gamma(x \cdot c_\Gamma y) = c_\Gamma x \cdot c_\Gamma y$,
(C') $c_\emptyset x = x$.

In analogy with the representation theory for cylindric algebras we introduce non-commutative diagonal-free set algebras by modifying 1.1.5 of HMT[5].

DEFINITION 1.4. An algebra $\mathcal{B} = \langle B, \cup, \cap, \sim, 0, 1, C_{(\Gamma)} \rangle_{\Gamma \subseteq \Omega}$ is a *strong* $NCDf_\Omega$ *set algebra* if there exists a sequence of sets $V = \langle V_a : a \in \Omega \rangle$ such that B is a nonempty subset of $Sb(\prod_{a \in \Omega} V_a)$ which is closed under the Boolean operations, contains $0 = \emptyset$ and $1 = \prod_{a \in \Omega} V_a$, and is closed under the generalized cylindrification $C_{(\Gamma)}$ for each $\Gamma \subseteq \Omega$ where

$$C_{(\Gamma)} X = \left\{ y \in \prod_{a \in \Omega} V_a : \exists x \in X \; x|_{(\Omega \sim \Gamma)} = y|_{(\Omega \sim \Gamma)} \right\}.$$

Let $\mathcal{B}[\Omega, V]$ denote the full set algebra with universe $Sb(\prod_{a \in \Omega} V_a)$ determined by Ω and V.

Two other notions from HMT are needed — the notion of a relativized algebra (cf. 2.2.1 of [5]) and the notion of a rectangular element (cf. 1.10.6 of [5]).

If $\mathcal{B} = \langle B, +, \cdot, -, 0, 1, c_\Gamma \rangle_{\Gamma \subseteq \Omega}$ is a strong $NCDf_\Omega$ and $b \in B$, the *relativized algebra* obtained from $\mathcal{B}$ and b is the algebra $\mathcal{R}l_b \mathcal{B} = \langle B', +', \cdot', -', 0', 1', c'_x \rangle_{\Gamma \subseteq \Omega}$ where $B' = \{x \cdot b : x \in B\}$ and for $x, y \in B$, $x +' y = x + y$, $x \cdot' y = x \cdot y$, $-'x = b \cdot -x$, $0' = 0$, $1' = b$, and $c'_x x = b \cdot c_\Gamma x$. In a similar way relativized algebras of other types of systems can be defined.

The following example indicates the connection between a relativized set algebra and the approximation algebra $\mathcal{B}_S$ derived from an information system S.

EXAMPLE 1.5. Suppose $S = \langle U, \Omega, V, f \rangle$ is an information system where f is a one-one function. Let $b = f(U)$ $(= \{fx : x \in U\})$. Then $\mathcal{R}l_b \mathcal{B}[\Omega, V]$ is isomorphic to $\mathcal{R}d^\varrho \mathcal{B}_S$, the ϱ-reduct of $\mathcal{B}_S$, where the operation $C_{(\Gamma)}$ corresponds to $\overline{\Omega \sim \Gamma}$ (cf. 2.6.1 of HMT[5]).

An element x in a strong $NCDf_\Omega$ $\mathcal{B}$ is *rectangular* if $c_\Gamma x \cdot c_\Delta x = c_{\Gamma \cap \Delta} x$ for all $\Gamma, \Delta \subseteq \Omega$. With the terminology above we can give a cylindric algebra version of Theorem 1.2.

THEOREM 1.6. *Every simple, complete atomic strong* $NCDf_\Omega$ *with rectangular atoms is isomorphic to a relativized, strong* $NCDf_\Omega$ *set algebra.*

P r o o f. Suppose $\mathcal{B}$ is a strong $NCDf_\Omega$ as in the hypothesis. Let $\mathcal{B}^* = \mathcal{R}d^\sigma\mathcal{B}$, the σ-reduct of $\mathcal{B}$, where the operation κ_P corresponds to $c_{\Omega\sim P}$. (Note that σ is the inverse of ϱ in 1.5.)

CLAIM. *$\mathcal{B}^*$ is a reduced knowledge approximation algebra.*

Condition (C′) implies that $\mathcal{B}^*$ is reduced, (A_0)–(A_3) are direct translates of (C_0)–(C_3) and (A_5) holds because the atoms of $\mathcal{B}$ are rectangular elements. Condition (A_4) is a consequence of $\mathcal{B}$ being simple. To see this we need two facts: (1) an ideal in a strong $NCDf_\Omega$ $\mathcal{B}$ is a Boolean ideal that is closed under c_Γ for all $\Gamma \subseteq \Omega$, and (2) $c_\Gamma x eq c_\Delta x$ whenever $\Gamma \subseteq \Delta$. [(2) follows from the rectangular atom condition.] By (1) and (2) it follows that if $\mathcal{B}$ is simple then $c_\Omega x = 1$ for all $0 \neq x \in B$; thus, (A_4) holds for $\mathcal{B}^*$ and the claim holds.

By 1.2, $\mathcal{B}^* \cong \mathcal{B}_S$ for some information system S and the construction gives f one-one. Hence, $\mathcal{B} = \mathcal{R}d^\varrho\mathcal{B}^* \cong \mathcal{R}d^\varrho\mathcal{B}_S \cong \mathcal{R}l_b\mathcal{B}[\Omega, V]$ by 1.5 as desired.

R e m a r k 1.7. Recently, Andras Simon has axiomatized the class of all subdirect products of relativized strong $NCDf_\Omega$ set algebras by a simple finite axiom schema of equations. For a detailed formulation of this equational generalization of 1.6 see the survey [8] by Németi (Theorem 8.1, Section 4).

2. Algebras of rough sets. In this section we follow the approach to rough sets formulated in [6] and [11]. A pair $U = \langle U, \theta\rangle$ that consists of an equivalence relation θ on a nonempty set U is called an *approximation space.* Every $X \subseteq U$ has an upper approximation $\overline{X}$ and a lower approximation $\underline{X}$ in terms of the θ-classes. Namely,

$$\overline{X} = \bigcup\{\theta x : x \in X\} \qquad \underline{X} = \bigcup\{\theta x : \theta x \subseteq X\}\,.$$

A *rough subset* of U is a pair $\langle \underline{X}, \overline{X}\rangle$ where $X \subseteq U$. We denote the collection of all rough subsets of U by $Sb_R(U)$ and let $\mathcal{P}_R(U) = \langle Sb_R(U), \vee, \wedge, {}^*, {}^+, 0, 1\rangle$ where $0 = \langle \emptyset, \emptyset\rangle$, $1 = \langle U, U\rangle$,

$$\begin{aligned}
\langle \underline{X}, \overline{X}\rangle \vee \langle \underline{Y}, \overline{Y}\rangle &= \langle \underline{X} \cup \underline{Y}, \overline{X} \cup \overline{Y}\rangle\,,\\
\langle \underline{X}, \overline{X}\rangle \wedge \langle \underline{Y}, \overline{Y}\rangle &= \langle \underline{X} \cap \underline{Y}, \overline{X} \cap \overline{Y}\rangle\,,\\
\langle \underline{X}, \overline{X}\rangle^* &= \langle U \sim \overline{X}, U \sim \overline{X}\rangle\,,\\
\langle \underline{X}, \overline{X}\rangle^+ &= \langle U \sim \underline{X}, U \sim \underline{X}\rangle\,.
\end{aligned}$$

In [11] it is shown that $\mathcal{P}_R(U)$ is a Stone algebra. Even more is true.

A *double Stone algebra* is an algebra $\mathcal{L} = \langle L, +, \cdot, {}^*, {}^+, 0, 1\rangle$ such that $\langle L, +, \cdot, 0, 1\rangle$ is a bounded distributive lattice, * is a pseudocomplement (i.e., $x eq a^* \Leftrightarrow x \cdot a = 0$), Stone's law holds (i.e., $a^* + a^{**} = 1$), $^+$ is a dual pseudocomplement (i.e., $x \geq a^+ \Leftrightarrow x + a = 1$), and the dual Stone law (i.e., $a^+ \cdot a^{++} = 0$) holds. A double Stone algebra is *regular* if $a^+ = b^+$ and $a^* = b^*$ imply $a = b$. See Grätzer [4], Beazer [1], and Katriňák [7] for basic facts about (double) Stone algebras.

An easy extension of the calculations in Theorem 1 of [11] shows

THEOREM 2.1. *$\mathcal{P}_R(U)$ is a regular Stone algebra for every approximation space U.*

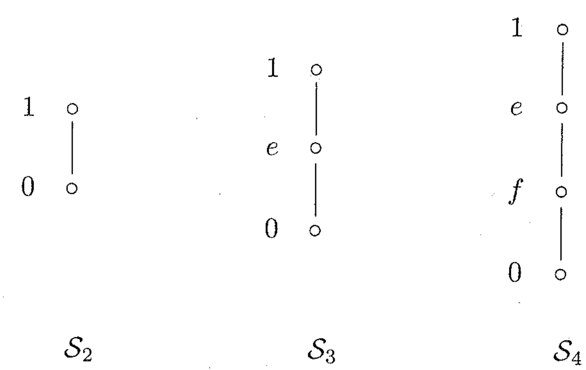
The main observation of this section is that the converse holds — every regular double Stone algebra has a representation as an algebra of rough subsets of an approximation space. First, we develop some notation and terminology.

If $\mathcal{L}$ is a double Stone algebra, let $C(\mathcal{L}) = \{x \in L : x^{**} = x\}$ denote the center of $\mathcal{L}$. $C(\mathcal{L})$ is a Boolean subalgebra of L on which * and $^+$ agree and are the Boolean complement. It is known that the congruence lattice $\mathrm{Con}(\mathcal{L})$ is distributive and that the congruences on a regular double Stone algebra correspond to normal filters of $\mathcal{L}$ — thus, the congruences commute (cf. [1]). These facts imply that

(I) the factor congruences on a double Stone algebra $\mathcal{L}$ form a sublattice of $\mathrm{Con}(\mathcal{L})$ that is a Boolean algebra (isomorphic to $C(\mathcal{L})$), and

(II) the congrunce generated by a proper filter on $C(\mathcal{L})$ is a proper congruence.

Let $\mathcal{S}_2$, $\mathcal{S}_3$, and $\mathcal{S}_4$ denote the 2-, 3-, and 4-element chains, respectively, considered as a double Stone algebra, i.e., $e^* = f^* = 0$ and $e^+ = f^+ = 1$ in the appropriate S_i.



In [7] Katriňák showed that $\mathcal{S}_2$, $\mathcal{S}_3$, and $\mathcal{S}_4$ are the only nontrivial subdirectly irreducible double Stone algebras. Note that $\mathcal{S}_2$ and $\mathcal{S}_3$ are regular, but $\mathcal{S}_4$ is not.

The following result may be known.

THEOREM 2.2. *Every regular double Stone algebra is a subdirect product of $\mathcal{S}_2$ and $\mathcal{S}_3$.*

Proof. Let $\mathcal{L}$ be a regular double Stone algebra. Because of (I) and (II) above, $\mathcal{L}$ is isomorphic to the algebra $\Gamma(X, T)$ of continuous sections of a reduced sheaf T over the Boolean space X of ultrafilters of $C(\mathcal{L})$ (cf. [2] or Section 3 of [12]). It follows that $\mathcal{L}$ is a subdirect product of the stalks T_x ($x \in X$). We claim that each T_x is isomorphic to either $\mathcal{S}_2$ or $\mathcal{S}_3$. To prove this, first recall that from the construction of the representation (cf. [2]) the algebra T_x is directly indecomposable, i.e., $C(T_x) = \{0_x, 1_x\}$. Also recall that in a directly indecomposable double Stone algebra, 0 is $\wedge$-irreducible and 1 is $\vee$-irreducible. Let T'_x denote the distributive lattice $T_x \sim \{0_x, 1_x\}$. We show that $|T'_x| eq 1$. Suppose $|T'_x| \geq 2$ for some x. Then T_x contains a subalgebra isomorphic to $\mathcal{S}_4$. Choose $\sigma, \tau \in \Gamma(X,T)$ so that $\{0_x, \tau(x), \sigma(x), 1_x\}$ is a subalgebra of T_x isomorphic to $\mathcal{S}_4$. Since X is a Boolean space and the elements of $\Gamma(X,T)$ are continuous there exists a clopen neighborhood N of x such that $\{0_y, \tau(y), \sigma(y), 1_y\}$ is isomorphic to $\mathcal{S}_4$ for all $y \in N$. Extend $\sigma|_N$ and $\tau|_N$ to $\mu, \nu \in \Gamma(X,T)$ such that $\mu \neq \nu$, $\mu^* = \nu^*$, and $\mu^+ = \nu^+$ (e.g., let $\mu|_{(X \sim N)} = \nu|_{(X \sim N)} = 0|_{(X \sim N)}$). This contradicts the regularity of $\mathcal{L}$ ($\cong \Gamma(X,T)$). Thus, $|T'_x| eq 1$ for all $x \in X$ from which it follows that T_x is isomorphic to $\mathcal{S}_2$ or $\mathcal{S}_3$.

Now we turn to the representation using algebras of rough sets.

THEOREM 2.3. *For sets I and J, $\mathcal{S}_2^I \times \mathcal{S}_3^J$ is isomorphic to $\mathcal{P}_R(U)$ for some approximation space U.*

P r o o f. Given I and J, set $U = I \cup (J \times 2)$ and let θ be an equivalence relation on U with θ-classes $\{i\}$ for $i \in I$ and $\{(j,0),(j,1)\}$ for $j \in J$. The rough subset $a = \langle I, I \rangle$ belongs to $C(\mathcal{P}_R(U))$. Thus, $\mathcal{P}_R(U) \cong \mathcal{R}l_a\mathcal{P}_R(U) \times \mathcal{R}l_a^*\mathcal{P}_R(U)$ and it is easy to verify that the relativized algebras $\mathcal{R}l_a\mathcal{P}_R(U) \cong \mathcal{S}_2^I$ and $\mathcal{R}l_a^*\mathcal{P}_R(U) \cong \mathcal{S}_3^J$.

Theorems 2.2 and 2.3 yield

COROLLARY 2.4. *Every regular double Stone algebra is isomorphic to a subalgebra of $\mathcal{P}_R(U)$ for some approximation space U.*

3. Algebras of rough relations. In this section we propose a set of axioms for the notion of a rough relation algebra. Basically the idea is to replace the Boolean condition in Tarski's axioms for relation algebras (cf. Section 5.3 of HMT[5]) with the axioms for a regular double Stone algebra. However, there are consequences of the Tarski axioms which hold in the standard model of rough relations, but whose derivation from the relation algebra axioms use the Boolean complement. For this reason additional axioms must be added to the modified Tarski list.

First, the standard model of an algebra of rough relations is described. If $U = \langle U, \theta \rangle$ is an approximation space, observe that $U^2 = \langle U^2, \theta^2 \rangle$ is also an approximation space.

DEFINITION 3.1. A *rough relation* on (an approximation space) U is a rough subset of U^2. The double Stone operations $\vee, \wedge, ^*$, and $^+$ introduced in Section 2 apply to the elements of $Sb_R(U^2)$. In addition, $Sb_R(U^2)$ contains the special element $1' = \langle \theta, \theta \rangle$ and the usual relational operations ; and $^\cup$ on $Sb_R(U^2)$ are defined coordinatewise, i.e., $\langle \underline{R}, \overline{R} \rangle ; \langle \underline{S}, \overline{S} \rangle = \langle \underline{R};\underline{S}, \overline{R};\overline{S} \rangle$ and $\langle \underline{R}, \overline{R} \rangle^\cup = \langle \underline{R}^\cup, \overline{R}^\cup \rangle$. The standard model $\langle Sb_R(U^2), \vee, \wedge, ^*, ^+, 0, 1, ;, ^\cup, 1' \rangle$ is denoted by $\mathcal{R}_U$. Subalgebras of an algebra $\mathcal{R}_U$ are called *algebras of rough relations.*

An abstract notion of rough relation algebra is introduced below. The proposed set of axioms should be regarded as tentative. As additional properties of the algebras $\mathcal{R}_U$ are discovered it may be desirable to augment the list.

DEFINITION 3.2. A *rough relation algebra*, a R^2A for short, is an algebra $\mathcal{A} = \langle A, +, \cdot, {}^*, {}^+, 0, 1, ;, {}^\cup, 1' \rangle$ which satisfies the following axioms:

(i) $\langle A, +, \cdot, {}^*, {}^+, 0, 1 \rangle$ is a regular double Stone algebra,
(ii) $(x \,;\, y) \,;\, z = x \,;\, (y \,;\, z)$,
(iii) $(x + y) \,;\, z = x \,;\, z + y \,;\, z$ and $z \,;\, (x + y) = z \,;\, x + z \,;\, y$,
(iv) $x \,;\, 1' = x = 1' \,;\, x$,
(v) $x^{\cup\cup} = x$,
(vi) $(x + y)^\cup = x^\cup + y^\cup$,
(vii) $(x \,;\, y)^\cup = y^\cup \,;\, x^\cup$,
(viii) $(x^\cup \,;\, (x \,;\, y)^*) \cdot y = 0$,
(ix) $(x \,;\, y) \cdot zeqx \,;\, x^\cup \,;\, z$,
(x) $x^{*\cup} = x^{\cup *}$ and $x^{+\cup} = x^{\cup +}$,
(xi) $(x^* \,;\, y^*)^{**} = x^* \,;\, y^*$,
(xii) $1'^{**} = 1'$.

A rough relation algebra is called *representable* if it is isomorphic to a subdirect product of algebras of rough relations.

The first observation concerning rough relation algebras deals with whether or not an ordinary relation algebra can be representable as a R^2A without being representable as a RA. For a set U, let $\mathcal{R}e(U)$ denote the relation algebra of all binary relations on U. Actually, $\mathcal{R}e(U)$ is isomorphic to $\mathcal{R}_U$ where the approximation space $\langle U, \theta \rangle$ has θ as the identity relation.

LEMMA 3.3. (i) *Every algebra of rough relations is a* R^2A,

(ii) *The center* $C(\mathcal{A})$ *of a* R^2A $\mathcal{A}$ *is a relation algebra that is a subalgebra of* $\mathcal{A}$,

(iii) $C(\mathcal{R}_U) \cong \mathcal{R}e(U/\theta)$ *for every approximation space* $U = \langle U, \theta \rangle$.

Proof. (i) and (iii) are routine; (ii) follows from 3.2(x)–(xii) and the fact that $C(\mathcal{A})$ is a Boolean subalgebra of $\mathcal{A}$.

THEOREM 3.4. *Every simple relation algebra that is representable as a* R^2A *is representable as a* RA.

Proof. If a simple relation algebra $\mathcal{A}$ is representable as a R^2A we may assume that $\mathcal{A}$ is isomorphic to a subalgebra of $\mathcal{R}_U$ for some approximation space $U = \langle U, \theta \rangle$. Since every $x \in A$ has a unique complement the image of $\mathcal{A}$ is a subalgebra of $C(\mathcal{R}_U)$ which is isomorphic to $\mathcal{R}e(U/\theta)$ by 3.3(iii).

Finally we observe that a construction for Stone algebras from [4] can be adapted to show that every relation algebra can be the center of a R^2A which properly extends it.

For a relation algebra $\mathcal{A}$ define $\mathcal{A}^{[2]} = \langle A^{[2]}, +, \cdot, ^*, ^+, 0, 1, ;, ^\cup, 1' \rangle$ where $A^{[2]} = \{\langle a, b \rangle \in A^2 : aeqb\}$, $0 = \langle 0, 0 \rangle$, $1 = \langle 1, 1 \rangle$, $1' = \langle 1', 1' \rangle$, $+, \cdot, ;$ and $^\cup$ are defined coordinatewise, $\langle a, b \rangle^* = \langle -b, -b \rangle$, and $\langle a, b \rangle^+ = \langle -a, -a \rangle$.

It is straightforward to verify

THEOREM 3.5. *$\mathcal{A}^{[2]}$ is a rough relation algebra and $C(\mathcal{A}^{[2]}) \cong \mathcal{A}$ for every relation algebra $\mathcal{A}$.*

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