Geometer's SketchPad Tutorial

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These notes are for a workshop that introduces teachers to the on the Geometer's SketchPad (version 4). They are a revision of previous notes designed for use with version 3. They have been used as part of a Modern Geometry course for teachers and can also be used as a self-directed tutorial. It consists of the following sections:

- <u>Introduction</u>. Covers basic Menu options and Tools; making a sketch.
- 2. <u>Animation and Tracing</u> Making sketches move.
- 3. <u>Custom Tools</u> Saving constructions so they can be replayed
- 4. <u>Measuring and Calculating</u> Using the Measure Menu to calculate angles and lengths
- 5. <u>Poincare Disk Model Scripts</u> Using hyperbolic scripts to investigate non-Euclidean geometry
- 6. <u>Transformations</u> Using reflections, rotations, and dilations
- 7. JavaSketchpad Documents
- 8. <u>Articles</u> about SketchPad

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Introduction

Starting the program

Go to *CitNov1 Programs* located in the *CitNet* folder on the desktop. Scroll through the list of programs to locate *Geometer's SketchPad*. Double click it.

Work area

The SketchPad window can be resized to suit your convenience. Each sketch has its own window inside the main SketchPad window. These can also be resized and several sketches can be active at the same time. See a Window at the right.



Menu items. They are <u>F</u>ile, <u>E</u>dit, <u>D</u>isplay,

<u>Construct</u>, <u>Transform Measure</u>, <u>Graph</u>, <u>Window</u>, and Help listed across the top. Their meaning will

change depending on the objects selected. Check each one to get an idea of the options available.

Tools (on left side):

The six basic tools do the following:

- Note: Note: Pointer: use to select and move objects
- Dot: use to create points
- Circle: hold the left mouse button and drag to create a circle
- Segment: hold the left mouse button and drag to create a segment (or line or ray)
- Text Tool: selecting this allows labels and text boxes to be created. The cursor changes to a finger.
- B. Solid Arrow: this tool allows you to create and use custom tools.

Note: pressing the left mouse button while over the segment tool allows it to be changed to a line or ray tool.

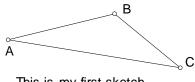
Next two simple figures will be created to illustrate the use of various tools and menu items.

Example 1.

Start with a clean sketch. This example constructs a triangle, labels the parts and adds a text box. The resulting sketch is shown below.

- 1. After selecting the dot tool click the work area in 3 places creating 3 points.
- 2. Select all three points. [To do this use the arrow tool. Hold the shift key while clicking each point. While it does not matter in this case, sometimes the order in which the points are selected are important; for example when selecting points to form an angle.]
- 3. With the points selected go to the *Construct* menu and select *Segments*. The three points should now be joined to form a triangle.
- 4. Click somewhere to deselect the sides of the triangle. With the arrow tool selected hold the mouse on different parts of the triangle and move.
- 5. **Labels**. Select the Text Tool. The cursor will change to a finger. Click each vertex and side of the triangle to reveal the default label. Clicking an object a second time will hide the label. To change the name of a label, double click the label with the finger. A dialog box will appear in which the name can be changed.

- 6. **Text boxes**. With the Text Tool selected move the mouse (finger) to an area where the text is to be placed. Double click the left mouse button to create a box for text to be entered. Add to your sketch a box that says: *This is my first sketch*. To reposition or resize the text box use the arrow tool. **Note:** when a text box is open for editing an array of font types and symbols is available at the bottom of the SketchPad window.
- 7. To delete an object or group of objects, select them and press the delete key. Be careful with delete. If you delete an object, its "children" (objects whose construction depends on them) will also go. If you make a mistake remember *Undo* is in the *Edit* menu. Instead of deleting an object it may be better to just hide it. This is illustrated in the next example.

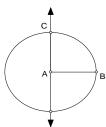


This is my first sketch.

Example 2.

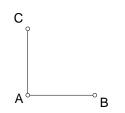
Start with a clean work area. In this example you will construct a line segment congruent to and perpendicular to a given line segment.

- 1. Select the <u>Segment</u> tool and draw a short line segment near the center of the work area. Label the endpoints A and B.
- 2. Select the point A and the segment (see step 2 in Example 1 if you forgot how). From the *Construct* menu choose *Perpendicular line*. This produces a line through A perpendicular to the segment.
- 3. Now select A and the segment again. This time choose *Circle by Center and Radius* from the *Construct* menu. This produces a circle with center A and radius AB.
- 4. Select the circle and the line. From the *Construct* menu choose *Point at Intersection*. The two points where the circle and line intersect are created.
- 5. Select one of the points created in Step 4 (call it C) and select A. Create the line segment AC using the *Construct* menu. You won't be able to see it because the original line perpendicular to AB covers the segment.



Clean up time! Select the <u>line</u> created in Step 2 and the circle created in Step 3. From the *Display* menu choose *Hide Objects*. Oh, the extra point created in Step 4 is still around. Hide it too.

Here is the result after clean up.

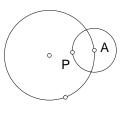


Now we have a segment AC congruent to AB that is perpendicular to AB. Select various parts and move. Notice that moving an initial point (A or B) changes the size or position, but moving a *child* (C or a segment AB or AC) moves the whole configuration.

Exercise: Continue the above construction to form a square that always retains its shape when moved.

Animation and Tracing Techniques

We need something to work with. Construct a circle, an arbitrary point A on the circle object, and a point P interior to the circle. Then construct a circle by center (A) and point (P) as in the picture.

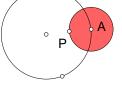


Constructing Interiors

First select the object, for example a circle or a series of vertices of a polygon. Then go to the **Construct** menu and select **Circle Interior** (or **Polygon Interior**).

Under the **Display** menu choose **Color** to set a color for the interior if you do not like the default choice.

Here is the result.



Animation

We want to animate the small circle with center A by moving A around the larger circle. First select the object(s) to animate: in this case the point A. It is constrained by the large circle. Having made these selections go to **Display**>**Animate Point**. The following Motion

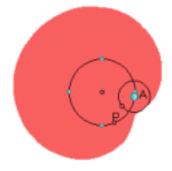
Controller box appears.

Motion Controller		×
Target: Point A 🛛 💌		
		4
Speed: 1.0		п

You can stop, start, pause, reverse direction, and change speed of the animation with this box. Closing the Motion Controller hides it without stopping the animation. To stop the animation without the Motion Controller select **Display**>**Stop Animation**.

Tracing

Animating an object is interesting but it would be nice to see the locus of points being generated. To do this you must turn on "Trace" before starting the animation. To turn the "Trace" feature on, select the object you want to trace. Then go to **Display>Trace**. That's it. Now when an animation (or any other movement) is performed the locus of the object will remain on the screen until the screen is clicked. Below is the result of tracing the circle interior when A animates around the circle.



Custom Tools

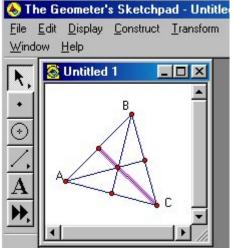
When investigating problems in SketchPad many constructions are performed over and over. GSP 4 allows *Custom Tools* (called *Scripts* in previous editions) to be created to eliminate the repetitive constructions. A Custom Tool can be played whenever that construction is needed. Below we see how to create a Custom Tool that is associated with a document. Also, we will indicate the process for placing a Custom Tool in a menu so that it is available for any document. More information can be found by going to the GSP **Help** item by searching for **Custom Tools**.

Creating a Custom Tool

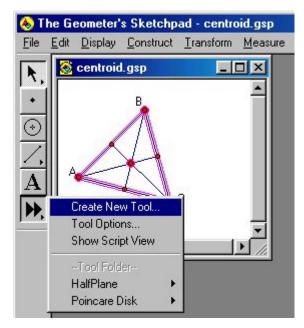
We will build a tool to construct the centroid (intersections of the medians) of a triangle. The strategy is to create a Tool by example. From the **File** menu select **New Sketch** if you do not have a clean document.

Start by constructing the centroid of a triangle. [That is creat three points (the triangle vertices), construct the segments connecting the vertices, construct the midpoints of each side, construct the segment from each midpoint to the opposite vertex, and finally construct the point of intersection of two of the medians (the third will fall on the intersection).]

Choose **Save As** from the **File** menu and save this sketch as **centroid.gsp**.

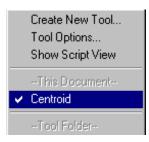


So far there has been nothing different from what we've done before. When we create a Tool we must specify the **Given** and the **Results**. For our Tool the Given will be the vertices of the triangle and the Results will be the sides of the triangle and the Centroid point. Click off the sketch to make sure nothing is selected then, in order, select the three vertices, the three sides and the Centroid point. Now click the left mouse button on the little arrow at the bottom right of the Custom Tool icon \mathbb{M} . This brings up the Custom Tool menu



Select **Create New Tool** and fill in the name "Centroid" and click "OK". **SAVE** the sketch.

Now, the Custom Tool menu has another item: Centroid. Before we use this Tool click on the option to **Show Script View**. You should see three initial vertices identified as **Given** and the **Steps** for the construction of the Centroid point.



Applying a Custom Tool.

Minimize the sketch used to create the Centroid Tool and open a new sketch. Create a triangle, create the midpoints of each side, and then join each of the three midpoints with segments. This should divide the original triangle into 4 (congruent) triangles. Now, we want to apply the Custom Centroid Tool to each of the 4 smaller triangles.

To do this, select Centroid from the Custom Tool menu. Notice the cursor changes to an arrow and has a point at the head (because the first Given is Point A). Also, notice the message at the bottom Geometer's Sketchpad that says: "Match Point A". Click one of the vertices of the triangle. Now, the message says: "Match Point B". Click on another vertex. The message say, "Match Point C", so do it. Notice that a new point – the centroid of the triangle whose three vertices you selected has now been created.

On your own

use the Centroid Tool to (1) find the centroid of each of the smaller triangles, (2) find the centroid of the large triangle, and (3) find the centroid of the triangle whose vertices are the centroids of the 3 "outside" triangles. How does this point relate to the others?

The Custom Tool folder

When we created the Centroid Tool above it appeared in the Custom Tool menu in the "This Document" section. To use such a Tool the document must be open – although not the current sketch. After having created a lot of Tools this can create quite a management problem. By placing a Custom Tool in a special folder, called the **Tool Folder**, the tool becomes available whenever Sketchpad is open without having to open the sketch used to create it.

The **Tool Folder** is located in the same folder that contains the GSP executable. It should have been created when the program was installed; if not, create it and be sure to name it **Tool Folder**.

Here is the process for adding frequently used tools to the Tool Folder.

(1) Create or copy the Custom Tools you want to put together into a new document.

(2) Choose **Save As** from the **File** menu, navigate to the **Tool Folder**, and save with an identifying name.

(3) The next time you start Sketchpad the name you specified will appear in the Custom Tools menu under Tool Folder and can be used.

Document Pages

Another way to organize sketches is to group them in pages. When a document has more than one page a page tab appears at the bottom of the document window. To add pages or to name a page, select **Tool Options** from the Custom Tool menu. Select **View: Pages**. Here you can hide or show the page tabs, name pages, add pages of remove page.

Measuring and Calculating

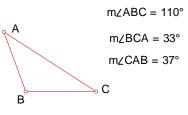
In some geometry problems it is necessary to measure angles or lengths and to perform calculations with the results. Tools for doing this (and more) are found in the **Measure** menu. The following examples should get you started.

The examples will use a triangle with vertices A, B, C. Start by constructing one.



Measuring angles

Select (using the shift-click method) vertices A, B, and C in order. The select **Measure** > **Angle**. The value m∠ABC should be placed on the sketchpad. You can position the equation wherever you wish by clicking it and dragging it to another location. Measure ∠BCA and ∠CAB in the same way.



Calculating

Ok, let's find the sum of the angles. Make sure nothing is selected in the sketch. Go to **Calculate** in the **Measure** menu. Clicking this brings up a calculator like screen with a black square in the center. Click the m \angle ABC equation. This moves m \angle ABC to the calculation window. Now, click "+" on the calculator keypad, then the m \angle BCA equation, the "+" again, and finally the m \angle CAB equation. Clicking "OK" places the calculation on the sketchpad. Notice that resizing the triangle causes the angle measures to change, but the sum of the angle measures stays the same. Hmmm!

Calculating lengths and distances

To calculate the length of the segment AB select the segment, then select **Length** from the **Measure** menu. To measure the distance from A to B select the two points A and B, then choose the **Distance** option from the **Measure** menu. What's the difference?

For practice calculate the lengths of the segments AC and BC. Then go to the Calculate option and compute AB^2+AC^2 and BC^2 . Move the vertices of the triangle until $AB^2+AC^2 = BC^2$. What's the key?

Other calculations

Area and perimeter: Before using these an interior needs to be selected. To form the interior of a polygon select the vertices in order then go to Interior under the Construct menu. Slope: Select a segment or line in order to use this option of the Measure menu. Equation: Select a line before using this option. Before using either slope or equation it makes sense to select Show Axes or Show Grid from the Graph menu. This way you see the coordinate system. From the Graph > Grid Form menu you can also select either Square, Rectangular, or Polar grid.

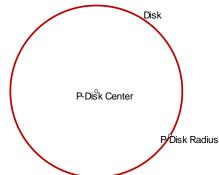
Hyperbolic Geometry Models

Models for hyperbolic geometry can be constructed in the Euclidean plane. Two such models are the <u>Poincare disk model</u> and <u>half-plane model</u>. Tools have been written for the Geometer's Sketchpad to help you work with these models. The scripts for the Poincare Disk model can be downloaded from **forum.swarthmore.edu/sketchpad/gsp.gallery/poincare/poincare.html** and the scripts for the half-plane model can be downloaded from

www.keypress.com/Sketchpad/misc/sibley/sibley.htm. Once the scripts have been downloaded they can be added as Custom Tools as in Section 3. These notes will only illustrate the use of tools created for the Poincare Disk model.

The basic model

Start with a blank sketch. Hold the mouse on the little arrow in the lower right-hand corner of the **Custom Tool** icon and select **Poincare Disk**, then **Poincare disk model** from the Tool Folder. (It is useful to turn-on **Show Script View** from the **Custom Tool** menu to see the order of the "givens" in the constructions.) To construct the basic Poincare disk (P-disk) model first click a point to fixes the center of the disk, then a second point to determine the radius. The labels can be changed as in the following example.



Your labels can be more concise, but it is important to identify the P-Disk Center and the P-Disk Radius. You will need these points to align other constructions to the model.

Using P-disk model tools

Why do we need the new tools and what do they do? To see the difference note that if you select two points in the P-Disk and draw a line through them using the standard GSP line tool, you get an Euclidean line, not the "P-line". To obtain the hyperbolic line, that is, an arc of a circle orthogonal to the circumference of the disk, additional constructions are needed. That's what the P-tools provide. Here is a list of the tools with a brief description of what each does.

Line	Constructs a Poincare line through two points
Segment	Constructs a P-segment between two points
Angle	Measures hyperbolic angle with second point as vertex
distance 2 points	Measures the (hyperbolic) distance between two points
perp bisector of segment	Constructs perpendicular bisector of P-segment
Pert Pt to line	Constructs perpendicular from point to P-line
angle bisector	Constructs bisector of angle determined by 3 points
Circle by center & pt	Constructs P-circle with given center passing through point
circle by center & radius	Constructs P-circle with given center and a radius specified
	by a segment
Perp Pt to Pt intersect	Constructs a line perpendicular to a point on given line determined by two points

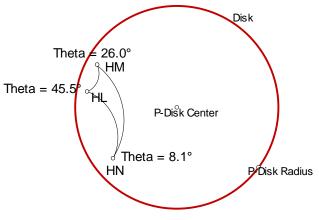
Illustration

Let's create a triangle in the P-Disk and measure an angle. Start with the basic Poincare model in a sketch.

Construct a P-triangle: Create 3 points in the interior of the P-Disk. Now select the **Poincare segment** tool from the **Custom Tool > Poincare Disk** folder. (Remember to show script view.) Click two of the three points, then the radius and finally the center of the disk. This creates a P-segment connecting the two selected points. On your own, construct to remaining two sides of the triangle. You should have something like the triangle below.



To measure an angle, select the Poincare angle tool from the **Custom Tool > Poincare Disk** folder. Notice the order of the points to be selected – the vertex of the angle is the middle point. Click the three vertices, then the disk radius and disk center. A textbox with the angle measure will appear in the upper left corner of the sketch. Right click the box and choose Properties if you want to change the default name of the angle. Repeat for the other two angles. Here is my result.



We conclude this section with a description of the tools available for the half-plane model.

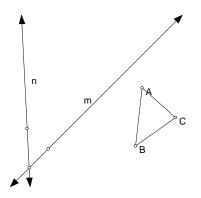
To work with the half-plane model first construct two points, P_1 and P_2 (from left to right) and an Euclidean line through them. The model will consist of the points above this line. Here is a list of the tools with a brief description of what each does.

Half-plane Line	Constructs the hyperbolic line through two points
Half-plane Segment	Constructs the hyperbolic segment through two points
Half-plane Angle	Measures hyperbolic angle with second point as vertex
Half-plane Distance	Measures the hyperbolic distance between two points

Tranformations

The Geometer's SketchPad has the capability to transform objects by translations, dilations (scaling), reflections, and rotations. These are found under the **Transform** menu. A full explanation of the transformations can be found under **Help** > **Menus**. Then choose **Transform menu**. Below reflections and rotations are illustrated.

To begin, construct a triangle with vertices A, B, C, two intersecting lines m and n, and the point H where the two lines intersect as pictured. (your labeling may be different.)



Reflection

We want to reflect triangle ABC about line m. First select the line m (the axes). Choose **Mark Mirror** from the **Transform** menu. Now, select the triangle and choose **Reflect** from the **Transform** menu. Congratulations! You just performed your first transformation. Select the "finger" tool and identify the labels of the new triangle. They should be A', B' and C'.

Now reflect the new triangle A'B'C' about the line n to get a triangle A''B''C''. You can see the results dynamically by moving one of the lines or the original triangle.

Rotations

Objects are rotated by a specified amount around a fixed point (the center of rotation). We will need to specify both of these. First check (and change if you so desire) the setting in the **Preferences** option of the **Edit** menu to see whether the unit for angle measurement is degrees or radians.

Select H as the center of rotation and choose **Mark Center** from the **Transform** menu. We want to mark an angle. Select a point on line m, H, and then a point on line n. Choose **Mark Angle** from the **Transform** menu. Now, we're ready to rotate. Select the triangle ABC and choose **Rotate** from the **Transform** menu. The dialog box should confirm you wish to rotate an object about H by the angle you have marked. (If you did not mark an angle, you will get a different dialog box which lets you enter the number of units for a rotation.) Click "OK' and you should see the results.

The results of the rotation should be selected (otherwise select it). Rotate the image one more time by the same amount around H. Where did the second image go? How does it compare with the two reflections? Move the lines and triangle to dynamically see if things change.

Conjecture a result you think is true about reflections and rotations.

Here is a quick introduction to dilations and translations.

Dilations

To dilate an object we need a center of the dilation (like an origin) and a scaling factor. The center is easy. Select a point and choose **Mark Center** from the **Transform** menu as you did with a rotation. The scale can be set in either of two ways.

<u>Numerically</u>: In this case select the object to be dilated and choose Dilation from the Transform menu. Then enter the numerical scaling factor in the dialog box and click "OK".

<u>Ratio of segments</u>: In this case first select two segments and choose **Mark Ratio** from the **Transform** menu. Then select the object to be dilated, choose **Dilation** from the **Transform** menu, and click "OK" to using the marked center and ratio.

Translations

A translation is specified by either giving a vector or by specifying the direction and magnitude of the translation. Both options are available here.

<u>Vector</u>: Select the points that determine the tail and the head of the vector (in that order). Then choose **Mark Vector** from the **Transform** menu. Now select the object to be translated and choose **Translate** from the **Transform** menu. In the dialog box make sure "Marked" is selected and click "Translate".

<u>Direction and Magnitude</u>: Select the object to be translated and choose **Translate** from the **Transform** menu. In the dialog box there is a choice of **Rectangular** (specify the x and y distances to be translated) or **Polar** (specify the direction $(0^{\circ}$ is the positive x-direction) and the magnitude of the translation). Fill in your choices, then click "Translate".

Note: After constructing a series of transformations you may want to have a single mapping that performs the composition. This single mapping can be defined as a Custom Tool (see Section 3).

JavaSketchpad Documents

This section provides instructions for how to convert an interactive sketch constructed with Geometer's SketchPad (version 4) into a Java applet for use via a browser. The resulting HTML file can be manipulated either over the web or locally from a disk.

Step 1. Organizing files

First decide where the HTML file and Java applets are to be placed. This may be in a folder on a web server (if to be accessed from the internet) or in a folder on a disk that can be given to students or placed on a stand-alone computer. Give the folder a name like **My JavaSketch files**. Now, into this folder <u>copy</u> the folder **jsp** that contains the applet class files. The **jsp** folder can be found where the Sketchpad files were installed, usually in the **Sketchpad** folder under **Program Files** on the C: drive. After copying the **jsp** folder into **My JavaSketch files** you are ready to begin.

Step 2. Create an interactive sketch

By now you have done lots of these. Save the sketch in the regular way so it can be used again.

Step 3. Saving the sketch as a HTML file

From the **File** menu choose **Save As** to bring up the Save As dialog box. In the "Save in:" field navigate to the folder **My JavaSketch files** so the HTML file will be saved in the same location as the **jsp** folder. Now, open the "Save as type:" box (the bottom one) and select the type: **HTML/JavaSketchpad Document** (*.htm). Now in the "File name:" box specify a name, e.g., sketch1. The extension should automatically be ".htm". If not, make sure the proper type is selected. Click the **Save** button and that's it.

Step 4. Test HTML file

You should be asked if you want to view the Java applet in the HTML file after you save. If not, exit GSP and go to the **My JavaSketch files** folder and double click the HTML file just created. Test to make sure it works.

Step 5. Cleaning up the HTML file

The JavaSketchpad applet created in the HTML file can be place on another web page or the created page can be edited to add additional information. The essential html code for the applet is

is

<APPLET CODE="GSP.class" ARCHIVE="jsp4.jar" CODEBASE="jsp" WIDTH=339 HEIGHT=279 ALIGN=CENTER> This is the code that makes construction work. </APPLET>

The applet tag can be copied to another page. Be sure the new page is located in the same place that the **jsp** folder is located. The CODEBASE specifies where a browser looks for the class files that make the applet work. The html code outside the <APPLET ... >... </APPLET> tag can be edited however you want it.

Articles that use Geometer's SketchPad

"An exploration of Brahmagupta's formula using The Geometer's Skrtchpad", Mary Beth Searcy, The Mathematics Educator, v. 4, no.2, Summer 1993, pp. 59-60.

"Technology in perspective", Albert Cuoco, E. Paul Goldenberg, and June Mark, <u>The</u> <u>Mathematics teacher</u>, v. 87, no. 6, September 1994, pp. 450-452.

"Teaching Relationships between Area and perimeter with The Geometer's Sketchpad", Michael Stone, <u>The Mathematics teacher</u>, v. 87, no. 8, November 1994, pp. 590-594.

"Characteristics of secondary geometry students' reasoning in the presence of a computer tool, the Geometer's Sketchpad", Gina Foletta, <u>Proc. 7th International Conference on technology in</u> <u>Collegiate Mathematics</u> November 1994, Addison-Wesley, 1996, pp. 164-168.

"Dynamic Geometry Environments: What's the Point?", Albert Cuoco, E. Paul Goldenberg, and June Mark, <u>The Mathematics teacher</u>, v. 87, no. 9, December 1994, pp. 716-717.

"The role of technology in mathematical modeling", Michael de Villiers, <u>Pythagoras</u>, No. 35, December 1994, pp. 34-42.

"From Paper Folding to the Algebraic Formula of Conics", Pao-Ping Lin, <u>The Mathematics</u> <u>Educator</u>, v. 5, no.1, Winter 1994, pp. 23-28.

Geometry Turned On, Doris Schattschneider and James King, Math. Assoc. America, 1997.

"Curves as Envelopes with the Geometer's Sketchpad", Alfinio Flores, <u>Mathematics and</u> <u>Computer Education</u>, v.31, no. 1, Winter 1997, pp. 56-65.

"Assessing Justification and Proof in Geometry Classes Taught Using Dynamic Software", Enrique Galindo, <u>The Mathematics Teacher</u>, v.91, no.1, January 1998, pp. 76-82.

"The Euler Line and Nine-Point-Circle Theorems", Frank Eccles, <u>The Mathematics Teacher</u>, v.92, no.1, January 1999, pp. 50-54

Rethinking Proof with The Geometer's Sketchpad, Michael de Villiers, Key Curriculum Press, 1999

"Exploring Hyperbolic Geometry with The Geometer's Sketchpad", Marlene Dwyer and Richard Pfiefer, <u>The Mathematics Teacher</u>, v.92, no.7, October 1999, pp. 632-637.

"Isoperimetric Quadrilaterals: Mathematical reasoning with Technology", Gina Foletta and David Leep, <u>The Mathematics Teacher</u>, v.93, no.2, February 2000, pp. 144-147.

"Using The Geometer's Sketchpad to Visualize Maximum-Volume Problems", David Purdy, <u>The Mathematics Teacher</u>, v.93, no.3, March 2000, pp. 224-228.

"Algebra in the Service of Geometry: Can Euler's Line Be Parallel to a Side of a Triangle?", Damon Diemente, <u>The Mathematics Teacher</u>, v.93, no.5, May 2000, pp. 428-431.

"The Angles of a Star", Alan Lipp, <u>The Mathematics Teacher</u>, v.93, no.6, September 2000, pp. 512-516.

"Posing questions from proposed problems: Using Technology to enhance mathematical problem solving", M. Santos-Trigo and E. Diaz-Barriga, <u>The Mathematics Teacher</u>, v.93, no.7, October 2000, pp. 578-580.

"A Triangle Divided: Investigating Equal Areas", Daniel Scher, <u>The Mathematics Teacher</u>, v.93, no.7, October 2000, pp. 608-611.